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On the estimation of the regime transition point in bubble columns

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Abstract

Using literature data of gas hold-up as a function of the superficial velocity from 16 different sources, a data bank of regime transition points was elaborated. It comprises 83 data related to a total of 20 systems, covering a wide range of physical properties and operating parameters, for both perforated and porous plate spargers. This data bank was employed to critically assess the quality of the predictions of the regime transition point given by available literature correlations. All correlations tested failed to provide a proper representation of the data bank, with rather high mean absolute deviations (always greater than 37%) and, in some cases, even physically inconsistent values were obtained. Thus, new empirical formulas were proposed for estimating the gas superficial velocity at the point of regime transition in bubble columns and the corresponding gas hold-up, whose mean absolute deviations were respectively equal to 17.7 and 21.1%.

Keywords: Regime transition; Bubble column; Flow regime; Correlation

1. Introduction

Within the broad class of gas–liquid contactors, bubble columns are certainly one of the most important members. Many applications of these units can be found in the chemical, petrochemical, pharmaceutical, food and environmental industries due to some intrinsic advantages. These include high heat- and mass-transfer rates, absence of moving parts, large interfacial areas, low operating and maintenance costs, little floor space requirements and the possibility of operation with solids without serious erosion or plugging problems [1,2]. Despite their simple construction and operation, bubble columns can be rather difficult to design and scale-up in view of their highly complicated hydrodynamics. Therefore, these units have attracted considerable attention and a vast number of both experimental and simulation studies on the topic can be found in the literature.

Depending primarily on the gas superficial velocity, u_G, a bubble column can basically operate under three different regimes [3,4]. For low u_G values, small, uniform-sized bubbles are observed, whose ascension trajectories are practically vertical (d_b <1–2 mm) or exhibit small-scale transverse and axial oscillations. There is little interaction between individual bubbles, with low breakage and coalescence frequencies. These are the characteristics of the homogeneous regime. At high gas superficial velocities, on the other hand, coalescence and breakage phenomena acquire importance, leading to a wide variety of bubble sizes, which characterises the heterogeneous regime. In this case, bubbles ascension trajectories are completely irregular, an almost-parabolic gas-hold-up profile develops and intense liquid circulation is observed. The change from homogeneous to heterogeneous regime is not a sudden event. On the contrary, it occurs gradually as the gas flow rate is raised. This results in a third operating regime between the other two extremes, the so-called transition regime.

For a given gas–liquid system, both the interfacial area and the transport coefficients in a bubble column are highly dependent on the prevailing operating regime [5–7]. In the case of most industrial units, operation at the heterogeneous regime is desired [1,8–10], but for some bioreactors the homogeneous regime is preferred [2,11]. Therefore, the prediction of the regime transition point acquires considerable importance.

The onset of regime transition in bubble columns is mainly a consequence of an increasing extent of bubble coalescence. Hence, the theoretical prediction of regime transition requires

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Nomenclature

a_{0-3}	empirical parameters in Eq. (10b)
A	channel cross-sectional area (m)
b_{0-3}	empirical parameters in Eq. (10c)
B	fluid-dependent parameter in Eq. (4)
Bo	Bond number
c_{0-5}	empirical parameters in Eq. (16)
C_v	virtual mass coefficient
d_b	bubble diameter (m)
d_{eq}	equivalent diameter of the sparger (m)
d_h	hydraulic diameter of a channel (m)
d_o	orifice diameter in the sparger (m)
D_c	column diameter (m)
D_s	sparger diameter (m)
e_{0-5}	empirical parameters in Eq. (16)
E	absolute prediction error (%)
Fr	Froude number
g	acceleration due to gravity (m/s^2)
H_b	bubbling height (m)
H_0	clear liquid height (m)
Mo	Morton number
n_o	number of orifices in the sparger
P	wetted perimeter of the channel (m)
Re	Reynolds number
u	superficial velocity (m/s)
U_b	bubble velocity (m/s)
We	Weber number

Greek letters

α	proportionality constant for the dispersion coefficient
ϵ	gas hold-up
ζ	porosity of the sparger
μ	viscosity (Pa s)
ρ	density (kg/m^3)
σ	surface tension (N/m)
φ	volume fraction in the system without bubbling

Subscripts

G	gas phase
L	liquid phase
max	maximal value
mean	mean value
min	minimum value
ref	reference value
S	suspended solids
trans	transition point

Superscripts

sw	swarm
∞	isolated bubble

the computation of the transient evolution of bubble size distributions in the column as a result of breakage and coalescence phenomena and its consequent effect on the flow pattern in the equipment. This is quite a challenging task, since hydrodynamics are fully coupled with breakage and coalescence frequencies. A few approximate flow regime maps are available, but none of them covers a wide range of industrial conditions [11]. Some theoretical transition criteria have even been developed [8,11–13], but, as detailed by Ribeiro and Lage [4], they all have limitations. In the past two decades, a considerable effort has been made to represent and understand the complex hydrodynamics of bubble columns with the aid of computational fluid dynamics (CFD) for a constant bubble diameter [14,15]. Depending on the adopted model formulation, a proper representation of each regime may already be achieved [16]. However, CFD models are still unable to predict the regime transition point. In order to do so, these models will have to account for bubble size distributions and their evolution. This, in turn, will require, apart from efficient numerical methods to couple Eulerian multiphase models with the population balance equation, reliable models for bubble coalescence and breakage frequencies. Upon reviewing the available models for bubble coalescence and break-up, Araujo [17] verified considerable discrepancy in the prediction of different models for a given condition, as well as physical inconsistencies in the classical models of Prince and Blanch [18] and Luo and Svendsen [19], which, so far, have been the most frequent choice in the simulation of bubble columns.

Consequently, in the near future, the prediction of the regime transition point will still have to depend upon empirical correlations. Compared to other important design parameters like the gas hold-up or the mass-transfer coefficient in the liquid phase, the point of regime transition has drawn considerably less attention in the literature when it comes to the development of correlations for its prediction. Moreover, there has apparently been only one attempt at comparing individual correlations and their prediction errors [20], and this was done with a somewhat limited data set. In the present contribution, a sound data bank was compiled from the literature to provide a rigorous assessment of the available correlations for predicting the regime transition point in bubble columns. The average deviations between experimental and predicted values were proven to be unacceptably high for all correlations and, therefore, new empirical relations were proposed.

2. Available relations for regime transition

A total of six empirical or semi-empirical relations was found in the literature for the prediction of the gas superficial velocity (u_{trans}) and/or the corresponding gas hold-up (ϵ_{trans}) at the point of regime transition in bubble columns. This number is surprisingly small if one considers that, in a recent review [4], more than 30 correlations were listed for the mass-transfer coefficient in the liquid phase and almost 40 correlations were given for the gas hold-up in this kind of gas–liquid contactor.

The oldest relation considered here is semi-empirical in nature and was developed by Kelkar [21] based on the concept of slip velocity and on the assumption of a parabolic radial distribution of gas hold-up:

$$u_{\mathrm{trans}} = 0.188U_{\mathrm{b}}^{\infty} + 0.333u_{\mathrm{L}} \quad (1)$$

In Eq. (1), the velocity of the isolated bubble, U_{b}^{∞}, is computed with the aid of the correlation presented by Grace et al. [22], assuming a bubble diameter equal to 6.0 and 1.0 mm for perforated and porous plate spargers, respectively.

During the development of a correlation for the gas hold-up that uses distinct relations for the homogeneous and heterogeneous regimes, Wilkinson et al. [23] obtained the following equations for estimating the transition point as a function of the physical properties of the gas–liquid system:

$$\epsilon_{\mathrm{trans}} = 0.5\exp(-193\rho_{\mathrm{G}}^{-0.61}\mu_{\mathrm{L}}^{0.5}\sigma^{0.11}) \quad (2)$$

$$u_{\mathrm{trans}} = 2.25\frac{\sigma\epsilon_{\mathrm{trans}}}{\mu_{\mathrm{L}}}\left(\frac{\sigma^3\rho_{\mathrm{L}}}{g\mu_{\mathrm{L}}^4}\right)^{-0.273}\left(\frac{\rho_{\mathrm{L}}}{\rho_{\mathrm{G}}}\right)^{0.03} \quad (3)$$

which are valid for pure liquids in the range $0.020 \le \sigma \le 0.073$ N/m; $4.0 \times 10^{-4} \le \mu L \le 0.055$ Pa s; $683 \le \rho_{\mathrm{L}} \le 2960$ kg/m^3 and $0.09 \le \rho_{\mathrm{G}} \le 38$ kg/m^3. This correlation is recommended by Deckwer and Schumpe [1] for design purposes.

Following the same ideas of Wilkinson et al. [23] but using different equations for the gas hold-up in each operating regime, Reilly et al. [24] utilised a total of 740 gas hold-up data to propose the following empirical equations:

$$\epsilon_{\mathrm{trans}} = 0.59B^{1.5}\rho_{\mathrm{G}}^{0.48}\sigma^{0.06}\rho_{\mathrm{L}}^{-0.5} \quad (4)$$

$$u_{\mathrm{trans}} = 0.352\epsilon_{\mathrm{trans}}(1-\epsilon_{\mathrm{trans}})\sigma^{0.12}\rho_{\mathrm{G}}^{-0.04} \quad (5)$$

in which B is a fluid-dependent parameter. The experimental data were related to 5 gases and 5 liquids and covered the following range of physical properties: $0.0235 \le \sigma \le 0.0728$ N/m; $5.75 \times 10^{-4} \le \mu_{\mathrm{L}} \le 2.433 \times 10^{-3}$ Pa s; $740 \le \rho_{\mathrm{L}} \le 1426$ kg/m^3 and $0.164 \le \rho_{\mathrm{G}} \le 1.84$ kg/m^3.

It should be noticed that Eqs. (2)–(5) were fitted based on gas hold-up profiles as a function of u_{G}. No comparison with actual values for the transition point was made. It was assumed that, since the gas hold-up profiles could be successfully represented with the proposed correlations for the whole u_{G} range investigated, the correlations for the transition point were appropriate. The predictions of Eqs. (2)–(5) were finally compared with experimental values of u_{trans} and $\epsilon_{\mathrm{trans}}$ by Krishna et al. [20]. Three different systems were considered and the correlation of Reilly et al. [24] was recommended. Nonetheless, such conclusion was drawn based on a limited data set, including a total of only 16 values for each variable.

Sarrafi et al. [25] correlated gas hold-up data from six different sources writing a different equation for the slip velocity in each operating regime. Reasoning that, at the transition point, both equations had to predict the same values, they ended up with the following implicit correlation for estimating the transition point:

$$\frac{u_{\mathrm{trans}}}{\epsilon_{\mathrm{trans}}} = U_{\mathrm{b}}^{\infty}\left[0.71 - 9\epsilon_{\mathrm{trans}} + 7.0\left(\frac{u_{\mathrm{trans}}}{U_{\mathrm{b}}^{\infty}}\right)^{0.75}\right] \quad (6)$$

$$\frac{u_{\mathrm{trans}}}{\epsilon_{\mathrm{trans}}} = U_{\mathrm{b}}^{\infty}\left[0.045 - 7.5\epsilon_{\mathrm{trans}} + 5.5\left(\frac{u_{\mathrm{trans}}}{U_{\mathrm{b}}^{\infty}}\right)^{0.5}\right] \quad (7)$$

for which U_{b}^{∞} should be estimated with the generalised correlation of Jamialahmadi and Müller-Steinhagen [26] based on the bubble diameter given by the model of Gaddis and Vogelpohl [27].

In a recent contribution, Thorat and Joshi [13] applied the theory of linear stability for the one-dimensional model of gas–liquid dispersion in the Euler–Euler framework to obtain an expression for the gas hold-up associated with the transition point:

$$\frac{U_{\mathrm{b}}^{\infty}}{\sqrt{d_{\mathrm{b}}g}} = \sqrt{\frac{\alpha(\epsilon_{\mathrm{trans}} + C_{\mathrm{v}})}{C_{\mathrm{v}}(1+C_{\mathrm{v}})}} \quad (8)$$

Fixed values were adopted for α, the proportionality constant for the dispersion coefficient, and, C_{v}, the virtual mass coefficient, whereas the bubble terminal velocity was obtained from experimental ϵ– u_{G} plots and then used to compute the corresponding d_{b} according to the recommendations of Clift et al. [28] for a contaminated system. A correlation for estimating C_{v} was later on presented by Bhole and Joshi [29]. However, it should be noticed that, without experimental ϵ– u_{G} plots, one cannot apply Eq. (8), whose prediction character is hence limited. This occurs because both d_{b} and the gas flow rate are unknown and, since no relation was given to compute u_{trans}, a bubble formation model to estimate d_{b} cannot be applied.

In the specific case of bubble columns operating with porous plate spargers, Kazakis et al. [30] computed the transition point based on their experimental ϵ– u_{G} data and then developed the following correlation for u_{trans}:

$$\frac{u_{\mathrm{trans}}^2}{d_{\mathrm{o}}g} = 0.023\left(\frac{D_{\mathrm{c}}^2\rho_{\mathrm{L}}g}{\sigma}\right)^{0.365}\left(\frac{D_{\mathrm{s}}}{D_{\mathrm{c}}}\right)^{1.825} \quad (9)$$

which is reported to predict the experimental data within 10% deviation.

3. Methodology

In order to enable a proper assessment of the predictive character of the equations presented in Section 2, experimental values of the gas superficial velocity and hold-up at the transition point for different systems are required. As reviewed by Vial et al. [31] and Shaikh and Al-Dahhan [11], many different methods have already been proposed for identifying the regime transition point in bubble columns. These are based on the analysis of either ϵ–u_{G} data sets or dynamic fluctuations of a signal related to the flow pattern (usually wall pressure).

In an attempt to keep all experimental data within similar precision values, a single identification procedure for all transition points included in the data bank was requested. The most popular method for regime identification, usually utilised to calibrate dynamic fluctuations methods, is the drift-flux analysis originally proposed by Wallis [32]. A considerable amount of experimental u_{trans} and $\epsilon_{\mathrm{trans}}$ values determined by the drift-flux method can be found in the literature. Nonetheless, there has unfortunately been a certain degree of confusion in the literature regarding the drift-flux method [3,33]. For the same experimental ϵ– u_G data set, different values of u_{trans} and ϵ_{trans} can be obtained, depending on the adopted model for the bubble swarm velocity (U_b^{sw}). Therefore, in the present contribution, instead of utilising reported experimental values for the transition point, which would inevitably be associated with different U_b^{sw} models, it was decided to take ϵ– u_G data from the literature and then calculate the corresponding transition point. One ought to realise that such a strategy substantially increases the number of candidates to integrate the data bank, for reported experimental values of u_{trans} and ϵ_{trans} are not required. Any of the numerous studies in which a ϵ– u_G data set is given could, in principle, be used.

As detailed in Table 1, a total of 83 data from 16 different sources [3,20,23,24,33–44] was collected, including both clear liquids and suspensions. Data were directly taken from the figures in the original papers using a software specifically designed for this purpose. Figures were either taken from PDF versions of the papers or scanned from printed versions. This procedure is believed to give a high precision of the reading, with an error lower than 1.0%. The effect of surface-active chemicals on regime transition was beyond the scope of the present contribution and hence data related to aqueous solutions of such substances were not included. In particular, this meant not including in the data bank any results associated with tap water, since it is known that trace impurities in the liquid phase have a major influence on gas hold-up and regime transition in bubble columns [45–47]. For each ϵ– u_G data set, the regime transition point was determined by two different methods, namely the bubble-swarm-velocity [48] and the drift-flux [32] methods, following the procedure described by Ribeiro and Mewes [33]. The mean of the individual values associated with each method was taken as the actual transition point for the corresponding ϵ– u_G data set.

A FORTRAN® code was written to compute the transition point with all equations whose predictive character was evalu-

Table 1
Sources of ϵ– u_G data used in the elaboration of the regime transition data bank

System	$D_c(m)$	Sparger	d_o (mm)	Operating range	Number of data	Reference
Air–water	0.15 and 0.29	Perf.	0.5	$0.25 \le H_0$ (m) ≤ 2	4	[3]
(Air, SF_6)–ethanol	0.05	Por.	0.03	$1.22 \le \rho_G$ (kg/m^3) ≤ 6.07	2	[20]
Nitrogen–(water, *n*-heptane)	0.15	Perf.	2.0	$684 \le \rho_L$ (kg/m^3) ≤ 6.07 $1.15 \le \rho_G$ (kg/m^3) ≤ 17.3 $0.41 \le \mu_L$ (mPa s) ≤ 1.0 $0.02 \le \sigma$ (N/m) ≤ 0.072	7	[23]
(Air, He, Ar, CO_2)–(water, isoparG)	0.15	Perf.	0.5	$740 \le \rho_L$ (kg/m^3) ≤ 1000 $0.20 \le \rho_G$ (kg/m^3) ≤ 4.47 $0.86 \le \mu_L$ (mPa s) ≤ 1.0 $0.02 \le \sigma$ (N/m) ≤ 0.07	11	[24]
Air–water	0.12	Perf.	0.7	$0.3 \le H_0$ (m) ≤ 0.6	4	[33]
Nitrogen–water	0.16	Perf.	2.0	$1.15 \le \rho_G$ (kg/m^3) ≤ 23.1	5	[34]
Air–(water, methanol, toluene, ligroin)	0.15	Perf.	2.3	$714 \le \rho_L$ (kg/m^3) ≤ 999 $0.47 \le \mu_L$ (mPa s) ≤ 1.0 $0.020 \le \sigma$ (N/m) ≤ 0.072	4	[35]
(Air, He, Ar)–(water, paraffin oil, tetradecane)	0.10 and 0.38	Por.	0.175	$763 \le \rho_L$ (kg/m^3) ≤ 998 $0.18 \le \rho_G$ (kg/m^3) ≤ 1.78 $1.0 \le \mu_L$ (mPa s) ≤ 2.3 $0.027 \le \sigma$ (N/m) ≤ 0.072	5	[36]
Air–(paraffin oil, parafin oil + silica)	0.10 and 0.38	Por.	0.05	$790 \le \rho_L$ (kg/m^3) ≤ 1249 $29 \le \mu_L$ (mPa s) ≤ 102 $0.05 \le \varphi_S \le 0.35$	7	[37]
Nitrogen–water	0.15	Perf.	0.5	$1.15 \le \rho_G$ (kg/m^3) ≤ 12.7	6	[38]
Air–water	0.14 and 0.15	Perf.	0.5	$\rho_L = 998$ kg/m^3	2	[39]
Nitrogen–Paratherm NF	0.10	Perf.	1.5	$\rho_G = 53.9$ kg/m^3	1	[40]
Nitrogen–Paratherm NF	0.05	Perf.	3.0	$846 \le \rho_L$ (kg/m^3) ≤ 892 $1.05 \le \rho_G$ (kg/m^3) ≤ 176.4 $10 \le \mu_L$ (mPa s) ≤ 50 $0.02 \le \sigma$ (N/m) ≤ 0.03	10	[41]
Nitrogen–(Tellus oil, glucose solutions)	0.15	Perf.	0.5	$867 \le \rho_L$ (kg/m^3) ≤ 1380 $9.21 \le \rho_G$ (kg/m^3) ≤ 11.5 $70 \le \mu_L$ (mPa s) ≤ 550 $0.03 \le \sigma$ (N/m) ≤ 0.08	3	[42]
Air–(water, water + calcium alginate beads)	0.14	Perf.	0.5	$998 \le \rho_L$ (kg/m^3) ≤ 1006 $1.00 \le \mu_L$ (mPa s) ≤ 2.73 $0.01 \le \varphi_S \le 0.30$	9	[43]
Air–(water, water + silica)	0.19	Perf.	0.5	$995 \le \rho_L$ (kg/m^3) ≤ 998 $0.98 \le \mu_L$ (mPa s) ≤ 1.27 $0.0005 \le \varphi_S \le 0.002$	3	[44]

ated in this work. In this computation, whenever the physical properties of the fluids were not given in the original reference, their values were estimated according to different methods. For pure liquids, the expressions from the DIPPR® data base [49] were used for all properties. The virial equation truncated to two terms, whose coefficient was computed with the correlation presented by Smith and Van Ness [50], was chosen for calculating the gas density. The effect of operating pressure on gas viscosity was modelled according to the method of Lucas [51], recommended by Reid et al. [52]. With regard to suspensions, their density was computed as the weighted mean of the values related to the individual phases using the volume fractions as weights, whereas, in the case of viscosity, the model of Krieger and Dougherty [53], recommended by Stickel and Powell [54], was utilised.

4. Results and discussion

4.1. Assessment of available correlations

Starting with the gas superficial velocity at which regime transition occurs, a comparison between the experimental u_{trans} values from the data bank and the corresponding predictions of the relations discussed in Section 2 is shown in Fig. 1. It is clear that none of the available correlations is able to provide a fair representation of the data set. Most of the values predicted by the expressions of Wilkinson et al. [23] and Sarrafi et al. [25] are much smaller than the experimental values, while the opposite trend is verified in relation to the recent correlation developed by Kazakis et al. [30]. In the case of the latter, the systematic deviation may be a consequence of the fact that the data used to develop the correlation were associated with a low sparger to column area ratio, being, therefore, more representative of a central bubble plume than a proper bubble column. Despite its somewhat better performance, the correlation of Reilly et al. [24] led to physically inconsistent values for the highest operating pressures considered, that is, the ones adopted by Lin et al. [41] in some of their experiments.

In order to put the general trend verified in Fig. 1, that is, the poor performance of the available correlations, into a more quantitative perspective, the mean, minimum and maximal absolute deviations for each correlation were evaluated and the results are listed in Table 2. Most equations could predict at least one of the experimental data with a deviation of less than 0.2%. Nonetheless, the mean deviations for the whole data bank are extremely high. The smallest value of E_{mean}, namely, 37.6%, is associated with the correlation of Kelkar et al. [21], but this is still too high for design purposes, not to mention the fact that the maximum deviation for the very same correlation can be as high as almost 450%.

The situation becomes even worse when it comes to the gas hold-up at the regime transition point. As evidenced in Fig. 2, regardless of the adopted correlation, only a small percentage of the data set is predicted within the ±30% error lines. In particular, the correlation of Reilly et al. [24], which was recommended by Krishna et al. [20], leads to $\epsilon_{\mathrm{trans}}$ values greater than 1.0 in the case of sufficiently high operating pressures. This stems from the direct dependence upon the gas density (see Eq. (4)) and explains, when one analyses Eq. (5), the negative values of u_{trans} previously shown in Fig. 1(c). The correlation of Wilkinson et al. [23], on the other hand, predicts a sheer drop in the gas hold-up with liquid viscosity and hence considerably underestimates $\epsilon_{\mathrm{trans}}$, giving, for high Morton numbers, $\epsilon_{\mathrm{trans}}$ values lower than 10^{-4}. Significant underprediction was also the main characteristic of the correlation of Sarrafi et al. [25], which proved to be little affected by the physical properties of the fluid phases.

For these three correlations, the absolute deviations are compared in Table 3. Due to the extremely high values of E_{mean}, in the case of $\epsilon_{\mathrm{trans}}$, there is little relevance in comparing the individual performances. Certainly, none of these equations can be recommended for a general estimation of the gas hold-up at the regime transition point. With regard to the correlation of Thorat and Joshi [13](Eq. (8)), it was not included in the comparison on account of its limited prediction character already explained in Section 2.

The reasons for the rather poor performance of the available correlations for estimating the regime transition point in bubble columns are believed to be twofold. Firstly, most of them were developed as part of a model for predicting the gas hold-up as a function of u_{G}. The successful representation of the ϵ curves was directly associated with the adequacy of the proposed relations for the transition point, which is not necessarily true. Secondly, the operating range covered in the present data bank is much wider than the application range of all correlations tested. Although true that one cannot guarantee the validity of a correlation outside its application range, the aim here is to assess the general prediction character of available relations for the transition point. It should be remembered that some of these equations have already been recommended for design purposes [1,20]. Therefore, relevant operating conditions for bubble columns, rather than specific application ranges for each correlation, seemed to be a more appropriate basis for the comparison.

Table 2
Absolute mean (E_{mean}), minimum (E_{min}) and maximal (E_{max}) prediction errors of available literature correlations for u_{trans}

Correlation	E_{mean} (%)	E_{min} (%)	E_{max} (%)
Kelkar [21]	37.6	0.05	448
Wilkinson et al. [23]	75.6	0.01	100
Reilly et al. [24]	42.7	0.19	581
Sarrafi et al. [25]	74.5	4.00	99.8
Kazaki et al. [30]	142	2.24	680

Table 3
Absolute mean (E_{mean}), minimum (E_{min}) and maximal (E_{max}) prediction errors of available literature correlations for $\epsilon_{\mathrm{trans}}$

Correlation	E_{mean} (%)	E_{min} (%)	E_{max} (%)
Wilkinson et al. [23]	75.8	1.61	100
Reilly et al. [24]	111.3	0.39	1528
Sarrafi et al. [25]	64.6	5.78	128.1

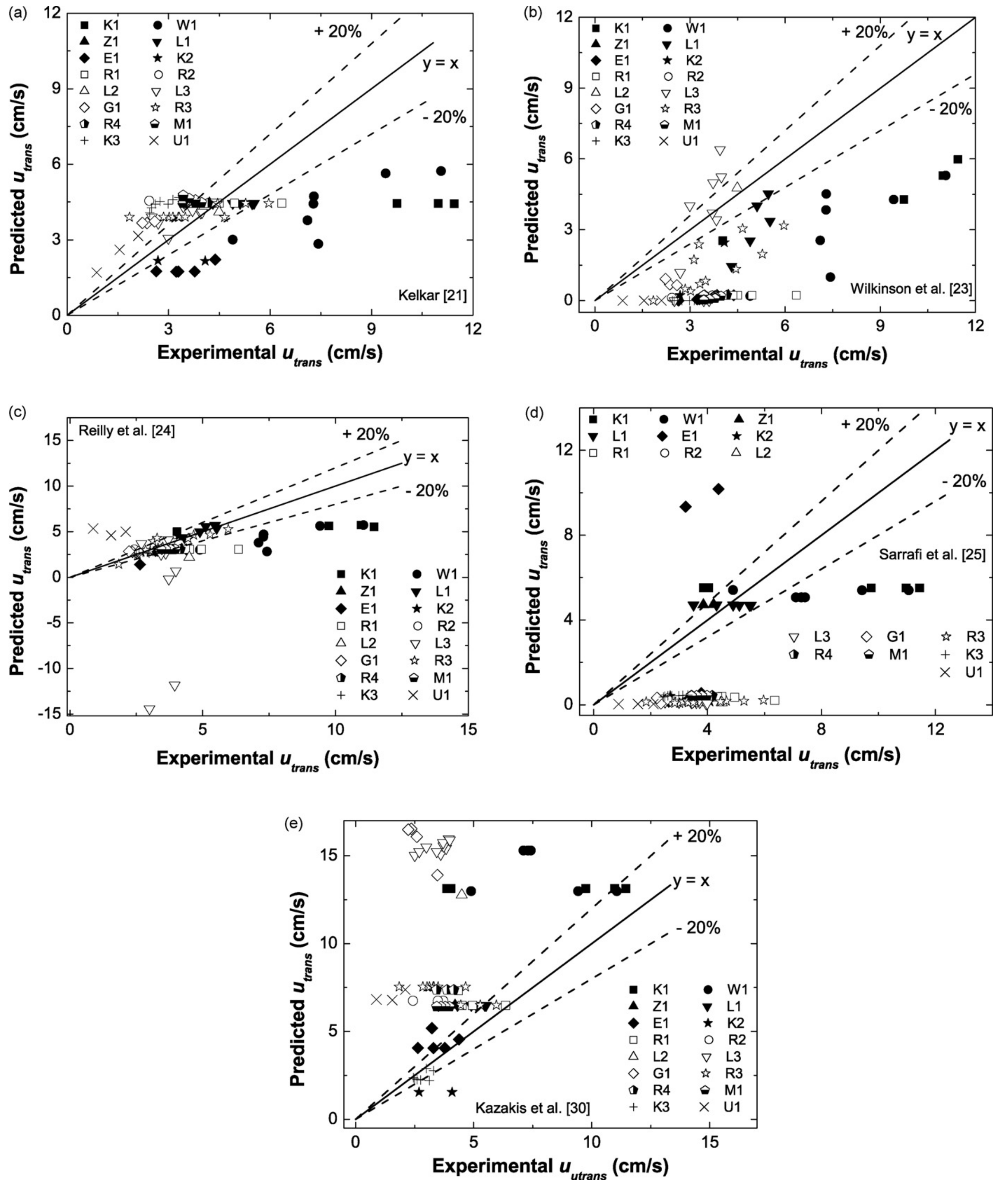

Fig. 1. Parity plots for the prediction of u_{trans} with different literature correlations: (a) Kelkar [21]; (b) Wilkinson et al. [23]; (c) Reilly et al. [24]; (d) Sarrafi et al. [25]; (e) Kazakis et al. [30]. Data sources: E1, Ellenberger and Krishna [36]; G1, Grund et al. [35]; K1, Krishna et al. [34]; K2, Krishna et al. [20]; K3, Krishna et al. [37]; L1, Letzel et al. [38]; L2, Luo et al. [40]; L3, Lin et al. [41]; M1, Mena et al. [43]; R1, Ruzicka et al. [3]; R2, Ruthiya et al. [44]; R3, Reilly et al. [24]; R4, Ribeiro and Mewes [33]; U1, Urseanu et al. [42]; W1, Wilkinson et al. [23]; Z1, Zahradnik et al. [39].

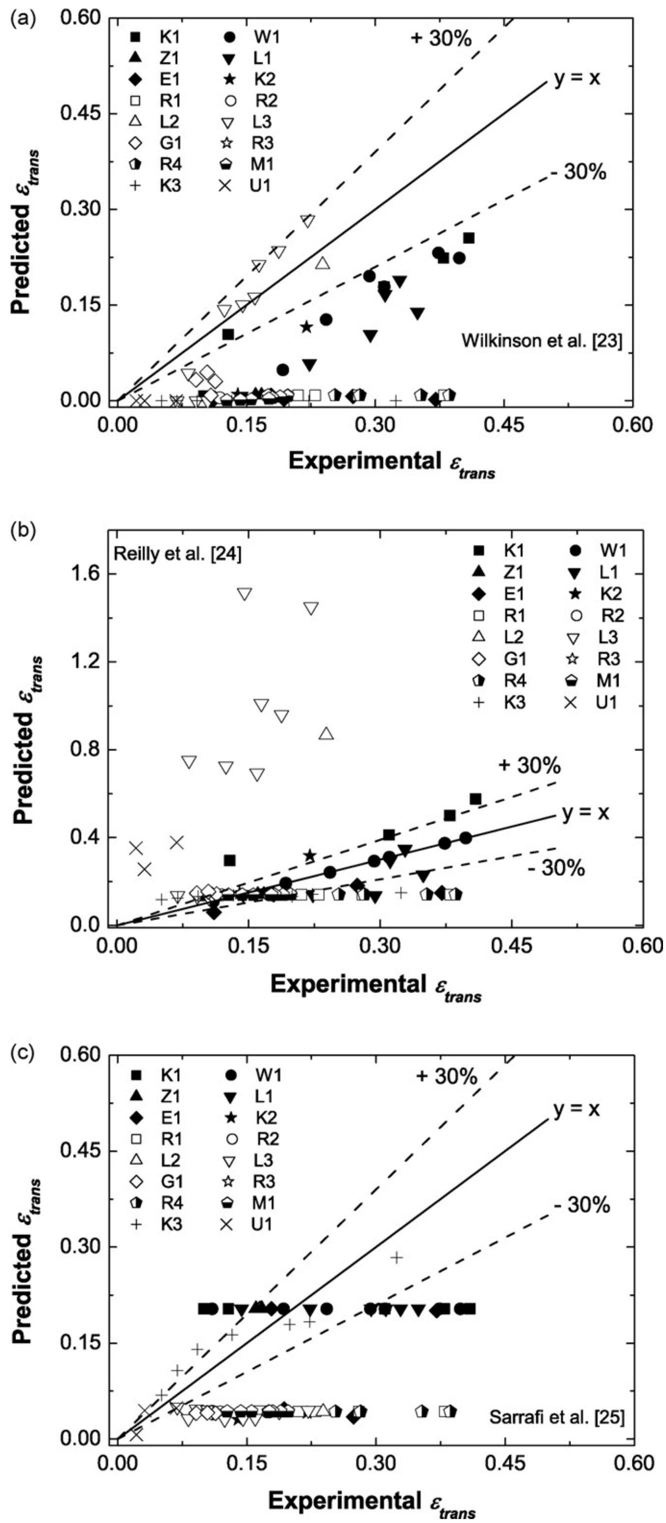

Fig. 2. Parity plots for the prediction of ϵ_{trans} with different literature correlations: (a) Wilkinson et al. [23]; (b) Reilly et al. [24]; (c) Sarrafi et al. [25]. The legend for data sources is the same adopted in Fig. 1.

4.2. Proposal of new relations

Tanking into account the high mean deviations observed for the available literature correlations, new empirical relations for estimating the regime transition point in bubble columns were proposed.

Initially, different expressions were sought to represent u_{trans} as a function of operating parameters chosen based on available experimental information about the relevant variables for regime transition and their respective effects [11,43,55–60]. The following equation, explicit in u_{trans}, was the one which provided the lowest mean deviation for the whole data bank:

$$\ln We_{\text{trans}} = f_1(d_{\text{eq}}, Mo) + f_2(d_{\text{eq}}, Mo, H_{\text{b}}, D_{\text{c}}) \ln Re_{\text{ref}} \quad (10a)$$

$$f_1(d_{\text{eq}}, Mo) = a_0 + a_1 d_{\text{eq}}^{a_2} + a_3 \ln Mo \quad (10b)$$

$$f_2(d_{\text{eq}}, Mo, H_{\text{b}}, D_{\text{c}}) = b_0 + b_1 \left(\frac{d_{\text{eq}} H_{\text{b}}}{D_{\text{c}}}\right)^{b_2} + b_3 \ln Mo \quad (10c)$$

whence

$$We_{\text{trans}} = \frac{u_{\text{trans}}^2 d_{\text{eq}}(\rho_{\text{L}} - \rho_{\text{G}})}{\sigma} \quad (11)$$

$$Mo = \frac{g\mu_{\text{L}}^4(\rho_{\text{L}} - \rho_{\text{G}})}{\rho_{\text{L}}^2 \sigma^3} \quad (12)$$

and Re_{ref} is a reference Reynolds number, computed with a reference value of gas superficial velocity, u_{ref}, given by a modification of the correlation of Reilly et al. [24]

$$Re_{\text{ref}} = \frac{u_{\text{ref}} d_{\text{eq}}(\rho_{\text{L}} - \rho_{\text{G}})}{\mu_{\text{L}}} \quad (13a)$$

$$u_{\text{ref}} = 0.352\epsilon_{\text{ref}}(1 - \epsilon_{\text{ref}})\sigma^{0.12}\rho_{\text{G}}^{-0.04} \quad (13b)$$

$$\epsilon_{\text{ref}} = \min(4.72\rho_{\text{G}}^{0.48}\sigma^{0.06}\rho_{\text{L}}^{-0.5}, 0.5) \quad (13c)$$

Despite its empirical nature, Eq. (10) is written in terms of independent variables that are physically related to bubble formation and coalescence phenomena. The Morton number is the standard dimensionless parameter to represent the effect of the physical properties of the liquid phase, found, for instance, in many correlations for gas hold-up in bubble columns [4]. Reynolds and Weber numbers, on the other hand, have already been adopted to understand and correlate the critical velocity for bubble coalescence [61–64].

In the definition of both Weber and Reynolds numbers, a characteristic length is required. Since a general correlation for both porous and perforated spargers is sought, the orifice diameter in the sparger would seem to be the natural choice. Nevertheless, though very important, the orifice diameter is not the only relevant parameter in the sparger that influences bubble formation. The total number of orifices must also be taken into account, because, for a given gas superficial velocity and orifice diameter, this parameter determines the gas flow rate per orifice, whose effect in the final bubble formation diameter can be rather significant [65]. Another important characteristic length for regime transition is the column diameter, which, in principle, could also be adopted for defining the dimensionless numbers. In fact, these three important variables can be combined into a single parameter, the equivalent diameter for the sparger, d_{eq}, with the aid of the classical definition of the hydraulic diameter of a general channel [66]:

$$d_h = \frac{4A}{P} \tag{14}$$

In the case of perforated plates, the application of Eq. (14) is straightforward, for the number of orifices is clearly defined and, therefore, $P = \pi(D_c + n_o d_o)$. For porous plates, an approximation of n_o can be computed based on the porosity of the sparger, namely, $n_o = \zeta(D_c/d_o)^2$. Substituting these expressions into Eq. (14), one gets the following formulae for d_{eq}:

$$d_{eq} = \begin{cases} \dfrac{D_c}{1 + n_o d_o/D_c} & \text{for perforated plates} \\ \dfrac{D_c}{1 + \zeta D_c/d_o} & \text{for porous plates} \end{cases} \tag{15}$$

As regards the gas hold-up, in order to represent the whole data set, two different empirical expressions were necessary depending upon the value of the Morton number, both dimensionless and explicit in ϵ_{trans}:

$$\epsilon_{trans} = \begin{cases} c_0 Bo^{-1} + c_1 Fr_{trans}^{c_2} (\rho_G/\rho_L)^{c_3} [H_b/(5D_c)]^{c_4} Bo^{c_5} & \text{for } Mo \le 10^{-9} \\ e_0 \ln Mo + Bo^{-1}\{e_1 + e_2 Fr_{trans}^{e_3} (\rho_G/\rho_L)^{e_4} [H_b/(5D_c)]^{e_5}\} & \text{for } Mo > 10^{-9} \end{cases} \tag{16}$$

where the dimensionless Bond and Froude numbers, chosen based on the work of Akita and Yoshida [67], are defined as

$$Bo = \frac{g d_{eq}^2 \rho_L}{\sigma} \tag{17}$$

$$Fr_{trans} = \frac{u_{trans}}{\sqrt{g d_{eq}}} \tag{18}$$

The empirical parameters in the proposed correlations, whose values are listed in Table 4, were fitted by non-linear regression with the aid of a commercial software using the regime transition data gathered in this work. It is important to highlight that, whenever the aspect ratio (H_b/D_c) was greater than five for a given operating condition, its value was set equal to five in the correlations. This is justified by the fact that, in the case of pure liquids, the effect of the bubbling height on regime transition is only observed for an aspect ratio smaller than five [13,14,55]. In all equations containing dimensional terms ((10b), (10c), (13b), (13c)), variables have to be expressed in SI units. The correlations are valid for pure liquids and suspensions within the wide range of physical properties and operating conditions covered in the data bank, that is:

$$2.11 \times 10^{-3} \le We_{trans} \le 1.94 \times 10^1$$
$$2.28 \times 10^{-11} \le Mo \le 1.47 \times 10^3$$
$$2.39 \times 10^{-3} \le Bo \le 6.40 \times 10^3$$
$$1.45 \times 10^{-2} \le Fr \le 1.66$$
$$2.34 \times 10^{-4} \le \rho_G/\rho_L \le 1.98 \times 10^{-1}$$
$$8.32 \times 10^{-5} \le d_{eq}(m) \le 1.35 \times 10^{-1}$$
$$H_b/D_c \ge 2.86$$

The comparison between the experimental values of u_{trans} and those predicted with Eq. (10) is presented in Fig. 3. Most of the data lie within the 20% error limits, with no physically inconsistent value, in a clear improvement of the pattern verified in Fig. 1 for previous correlations. In fact, the mean absolute deviation associated with Eq. (10) for the whole data set was only 17.7%, which is less than half the value of the smallest mean deviation obtained with the literature correlations tested.

Experimental and predicted values of ϵ_{trans}, in turn, are compared in Fig. 4. Even though the scatter is somewhat higher than in the case of the gas superficial velocity, the overall performance of the proposed correlation is still considerably superior to the one associated with any of the literature correlations tested, which becomes evident when one compares Figs. 2 and 4. Apart from the fact that no physically inconsistent value is obtained, the data are homogeneously distributed around the $y = x$line, without the tendencies of underestimation verified in Fig. 2(a) and (c). Moreover, the mean absolute deviation, in this case,

Table 4
Empirical parameters in the proposed correlations for estimating the regime transition point (Eqs. (10) and (16)).

Index	Parameters *a*	*b*	*c*	*e*
0	3.7976	1.4519	-1.3500×10^{-2}	-8.8958×10^{-3}
1	-1.2155×10^{1}	-8.4152×10^{1}	2.4176	-9.3307×10^{-4}
2	1.5160×10^{-1}	1.8264×10^{1}	6.7979×10^{-1}	1.2086×10^{-1}
3	3.7598×10^{-1}	-1.6063×10^{-2}	1.8012×10^{-1}	−2.0562
4	–	–	-6.1209×10^{-1}	7.5239×10^{-1}
5	–	–	7.5273×10^{-2}	−2.6870

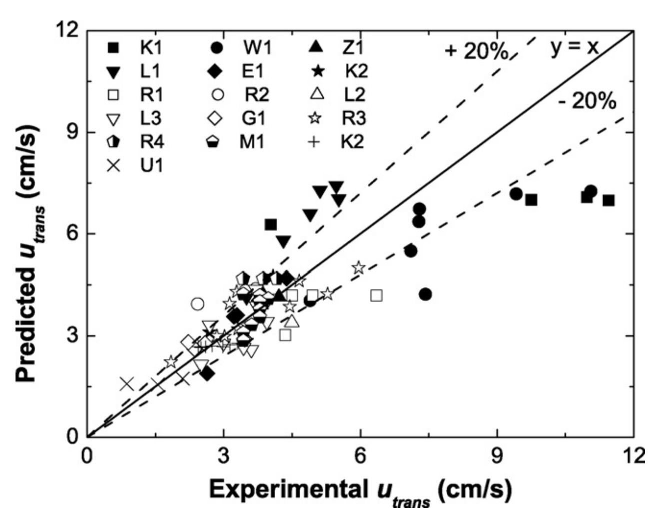

Fig. 3. Comparison between experimental values of the gas superficial velocity for regime transition and the predictions given by the correlation proposed in this work (Eq. (10)). The legend for data sources is the same adopted in Fig. 1.

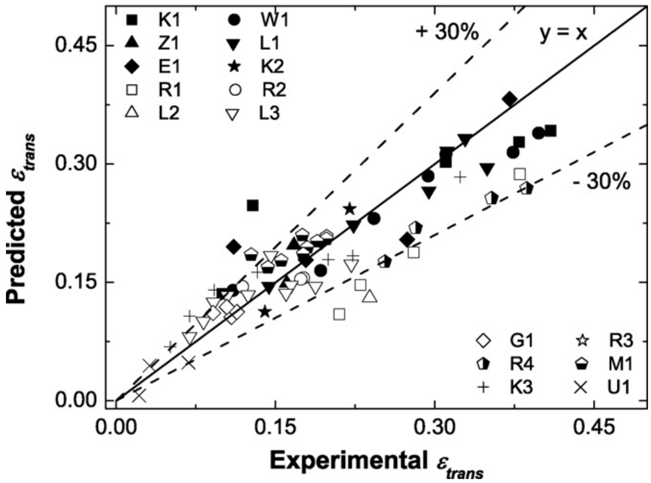

Fig. 4. Comparison between experimental values of the gas hold-up at regime transition and the predictions given by the correlation proposed in this work (Eq. (16)). The legend for data sources is the same adopted in Fig. 1.

was equal to 21.1%, a value which is lower than one third of the smallest mean deviation verified with previous correlations for ϵ_{trans}. Such mean absolute deviation has to be judged taking into account the wide range of operating conditions analysed and the fact that data from 16 different investigators were considered. Ribeiro and Lage [4] demonstrated, for instance, that mean deviations greater than 29% resulted from the application of eight different gas-hold-up correlations to a large data set from six different sources.

5. Conclusions

A large data bank of regime transition points in bubble columns under a wide range of operating conditions was elaborated using literature data of gas hold-up as a function of the gas superficial velocity. It was demonstrated that none of the correlations previously proposed for estimating the regime transition point could provide a satisfactory representation of the whole data bank, with mean absolute deviations always greater than 37 and 64% for the gas superficial velocity and gas hold-up, respectively. New empirical relations were proposed, valid for the whole data bank, whose mean deviations were lower than 22%. These relations represent an advance in comparison to previous equations and are therefore recommended for estimating the regime transition point in bubble columns.

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